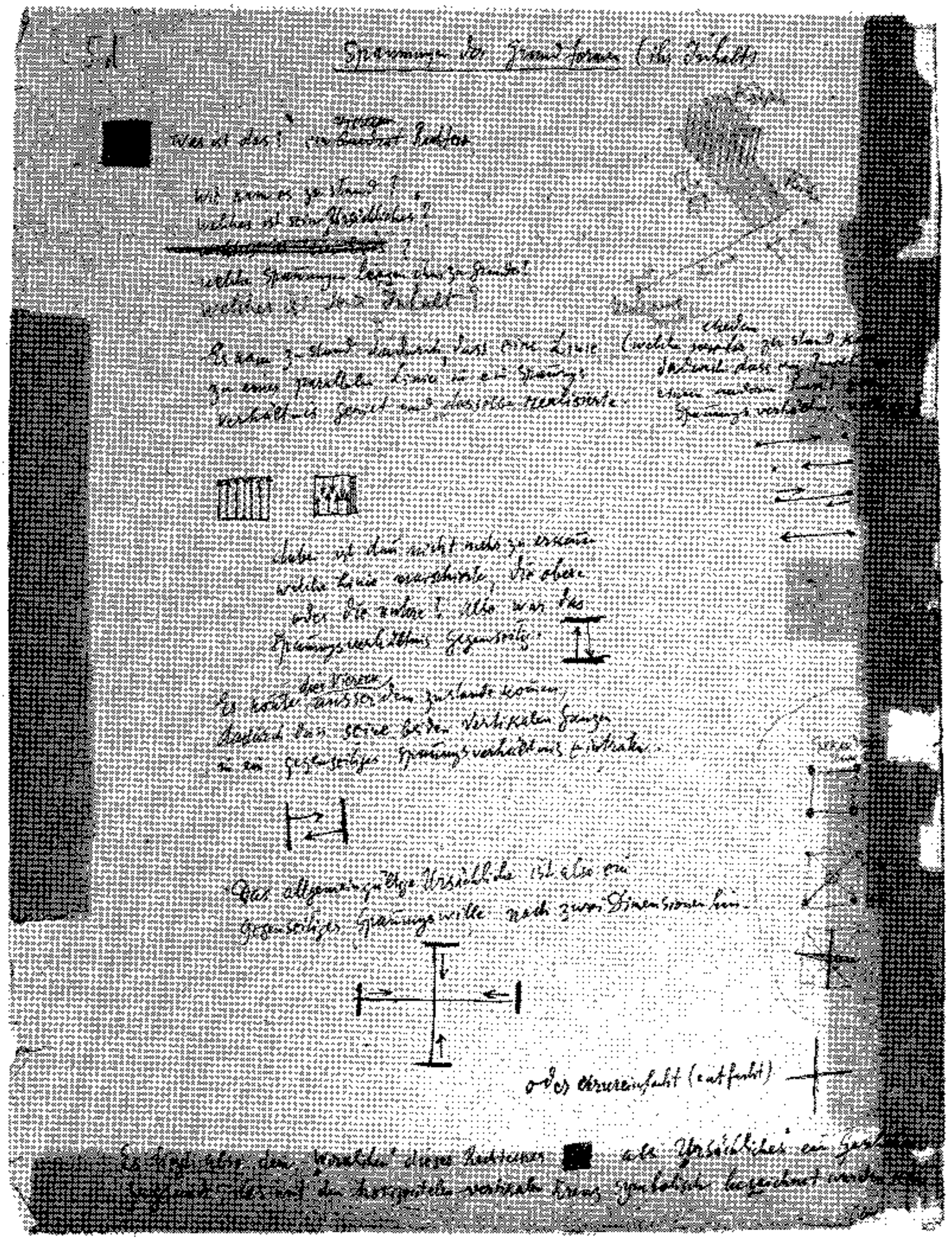


1.0
Paul Klee, "Point, Line, Plane, Volume," (upper right.) Paul Klee-Stiftung. Kunstmuseum, Bern, Switzerland.

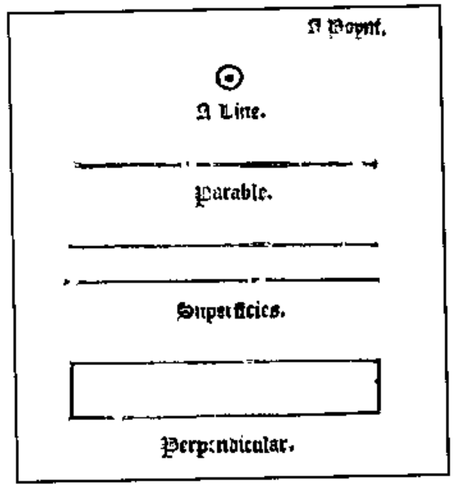
Chapter One

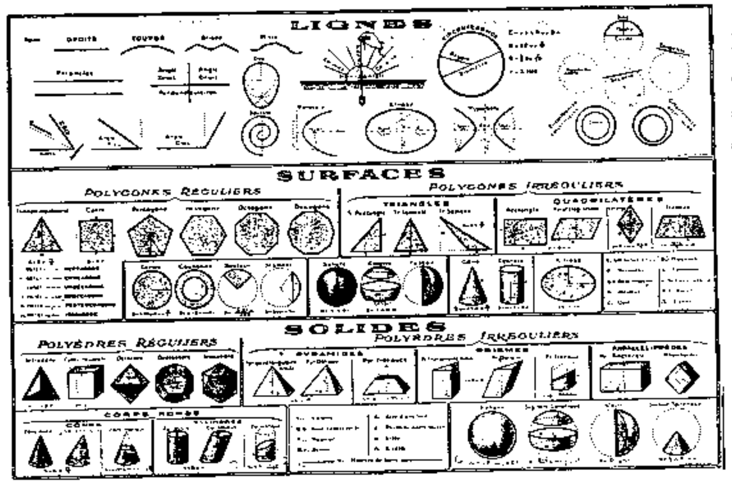
Basic Terms: Point, Line, Plane, Volume

Geometry is the means, created by ourselves,
whereby we perceive the external world
and express the world within us.
-Le Corbusier

Four centuries separate **Sebastiano Serlio's** (1475-1554) *Five Books on Architecture,* 1545 from **Le Corbusier's** (1887-1965) *The City of Tomorrow,* 1924. Yet both books begin in exactly the same way, with illustrations of the basic geometric elements. The two architects worked in remote time periods, designed buildings of widely diverse character, made use of different structural and constructional technologies, and related to very different political, social, and cultural pressures. Yet for both, simple geometry and the basic **Pythagorean progression** of point, to line, to plane, to volume constituted the fundamental ingredients of all architectural form.

Why were these terms useful to Serlio and Le Corbusier? Geometry did not provide specific solutions to questions of style, building program, or construction, since the two architects were as far apart as possible on these issues. Instead, both architects used geometry to establish an architectural order. A geometrically determined definition of form and arrangement of parts made it possible for both





1.1 *Left: Sebastiano Serlio, Five Books on Architecture, Book I, Plate 1, 1545.*

1.2 *Right: Le Corbusier, The City of Tomorrow, Frontispiece, 1924.*

architects to express architectural ideas clearly, and further, to allow for the development of sub-themes and variations which could be understood against the strong underlying formal order, or **datum**.

In discussing the role of geometry in architecture we shall begin, like Serlio and Le Corbusier, with a point. Equal in their relationship to all surrounding directions, points establish centers for groupings of form. While geometers define a point as dimensionless, in our discussion of architectural form we understand a point as any simple, singular thing: a pin point, a dot on a page, a sugar cube, a cylindrical oil drum, etc.

The line, mathematically defined, is a one-dimensional entity which can be infinitely extended in two directions. Rather than consider a line to be of a wholly different nature than the point, one can imagine a transformational relationship which links the two. In other words, a line can be understood as a point which has been dragged or 'translated' in space.

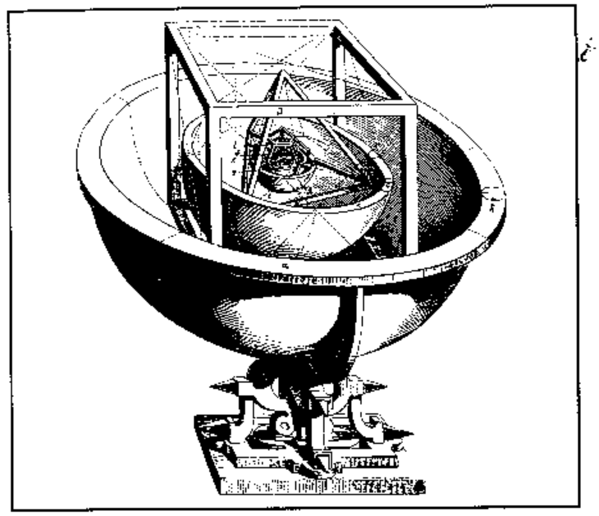
The next term in the Pythagorean progression is the plane, a two dimensional, unbounded surface which extends infinitely in all directions. How does a plane relate to the previous term, the line? Just as the idea of transformation helps us to understand the relationship between the point and the line, the same strategy can be used to clarify the relationship between the line and the plane. A plane can be understood as a line that has been dragged in space.

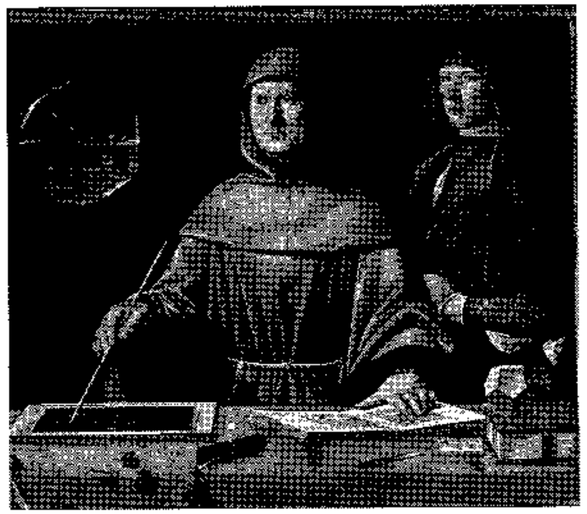
In discussing architecture, lines and planes do not have infinite extension: nothing in the physical world has that property. Still, the implication of extension gives power to the form, and establishes a potent demarcation between front and back, above and below, inside and outside, etc. Such distinctions permit further levels of spatial stratification to take place. Linear and planar structures can act in many different ways. As solid components of the built environment, walls separate one precinct from another; slabs establish boundaries between vertically stacked spaces; filters effect a transformation of spatial experiences from one side to the other; the neutral plane, or datum, acts as a background against which a free play of form can be deciphered.

As voids, linear elements often have even greater power to organize form. Axes can forge clear relationships across long distances without the requirement of adjacency; views or vistas make possible an understanding of connection between even more loosely grouped elements; paths orchestrate a sequence of experiences along the procession and enlist memory in the task of reconstructing and assembling the experience as an entirety.

The last term in the Pythagorean progression is the volume. Again, we can use our notion of transformation to understand the relationship between the three-dimensional volume and the preceding two-dimensional term. If a plane is dragged through space the resulting form is a cube or some other rectangular prism. Hence point, line, plane, and volume, the primary terms needed to establish an architectural order, are all related by a one simple act of transformation, as **Paul Klee's** (1879-1940) drawing, ***"Point, Line, Plane, Volume"*** illustrates.

Further affinities and distinctions can be found among these four basic elements. A line has only one more dimension than a point, yet





1.3 Left: Johannes Kepler, Harmony of the spheres, Mysterium Cosmographicum, 1621.

1.4 Right: J. de Barbari, 'Luca Paccioli,' (author of Divina Geometria), detail, c.1500, Capodimonte Museum, Naples.

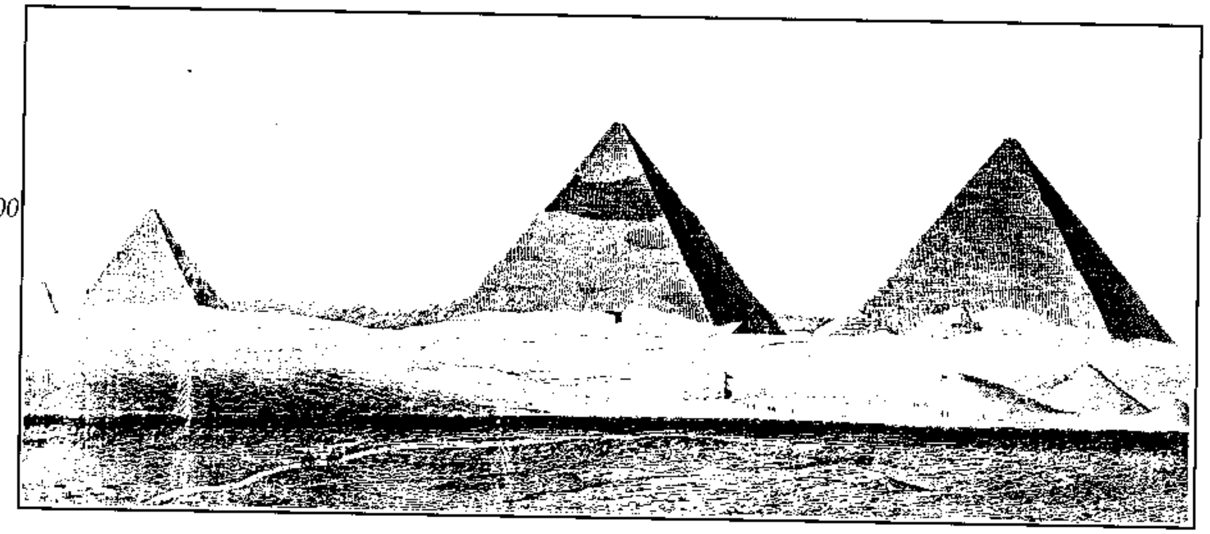
they function very differently in the making of space. The primary task of points is the definition of center, while the primary task of lines is the definition of edge or perimeter. Points are finite and bounded; lines are by definition extensible. Volumes and points, on the other hand, play similar roles in spatial organizations. Even though volumes are three dimensional and points are conceptually dimensionless they can be thought of as equivalent, but for the matter of scale. The farther we are away from something, the more it tends to have qualities of a simple, dimensionless point. When seen from the earth, stars appear to be minuscule points, although we know they are of vast size. When examined under a microscope, grains of sand appear to be volumetric, although under ordinary circumstances they appear as mere specks. Both points and volumes are bounded, finite and neutral towards the various directions which surround them. Hence, both can act as centers to organizations. On the contrary, lines and planes are defined both by extension and by their ability to set up distinctions between one side and the other. They do not act as centers but as edges to organizations of form.

Let us examine two structures which are bounded and crisply defined in form, an **Egyptian pyramid** from more than four thousand years ago, and a **cylindrical corn crib** from a Midwestern farm in the 1990's. Both are understandable as almost pure **Platonic solids**, or elemental three dimensional forms. The term 'Platonic solids' derives from the writing of Plato, the Greek philosopher who speculated that the four elements in nature, earth, air, fire and water, corresponded to geometrical solids whose faces are regular polyhedrons. For Plato, these were the tetrahedron, the cube, the icosahedron and the dodecahedron. The sphere was reserved as the shape of the heavens above. Believing in the cosmological significance of Platonic solids, **Johannes Kepler** constructed a model of the universe by nesting the Platonic

1.5
Cylindrical corn crib, Midwestern farm, 1990's.

forms around a center. **Luca Paccioli** wrote a treatise on the innate proportional relationships among simple solids in *Da Divina Proportione*. Both sought to reveal the underlying order in the **"Harmony of the spheres"** which governed relationships of everything from astral bodies to musical harmony.

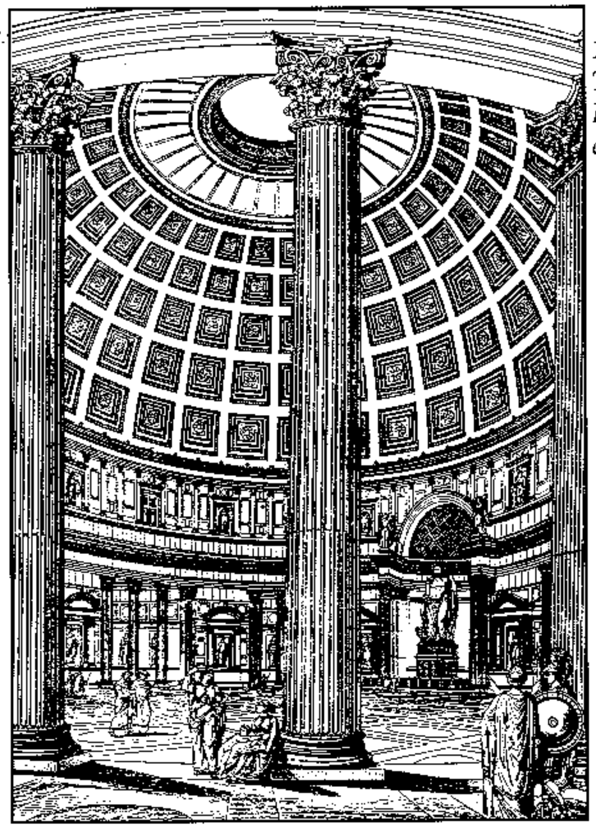
In every day discourse, 'Platonic forms' have come to mean volumes like spheres, cylinders, cones, pyramids, cubes, tetrahedrons, and the like which are generated when idealized two-dimensional forms (circles, squares, isosceles triangles, etc.) have been extended, rotated or reflected around their centers or axes. Both the cylinder and the pyramid, by their own mathematical definitions, make reference to their centers. Both are organized by a vertical **axis,** or conceptual center line, running from the ground through the top of the roof. Indeed, one symbolic purpose of the pyramid was to gather up the earth into a single point to meet the sun. Hence, at changing scales, the function of these



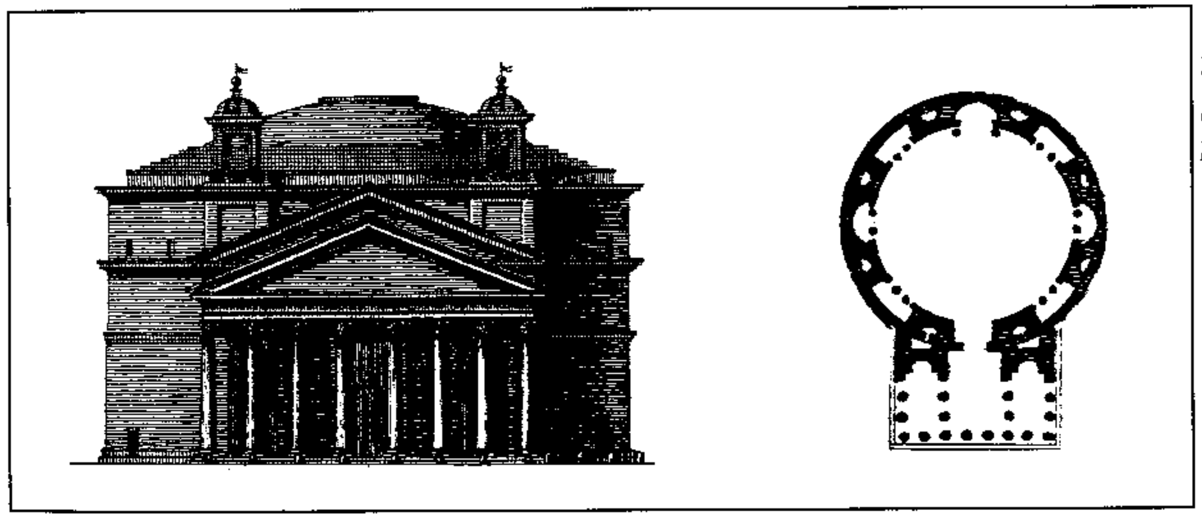
1.6
The Pyramids of Cheops (c. 2570 B.C.), Chefren (c. 2530 B.C.), and Mycerinus (c. 2500 B.C.) at Giza, Egypt.

structures as points or volumes is interchangeable.

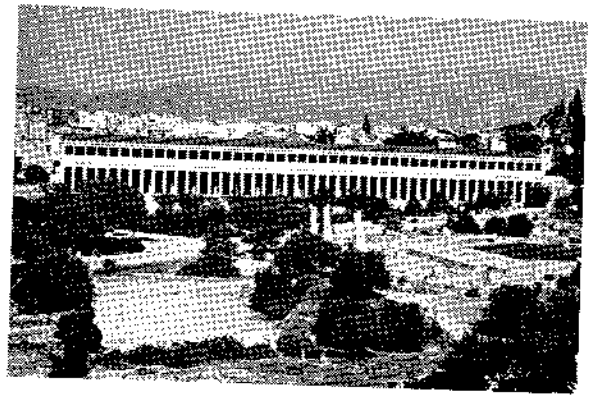
A more complex organization takes place in the next figure, the **Pantheon,** a circular Roman temple. Like the corn crib in the previous example, the Pantheon is essentially a domed cylindrical volume, but here the form is hybrid. A cylindrical drum is united with a temple front, joined together by a thick masonry block. Unlike the previous example in which all surrounding directions were addressed with equanimity, the addition of the temple portico introduces an axis into the centralized organization. The axis stems from the elongation of the structure in one direction, and the path of movement into the buildings. Even so, the axiality of the path is resolved in the vast centralized interior space. In the interior another axis is revealed: the **axis mundi**, or the vertical axis which links the earth to the sky, framed by the circular window or **oculus** at the top of the dome.



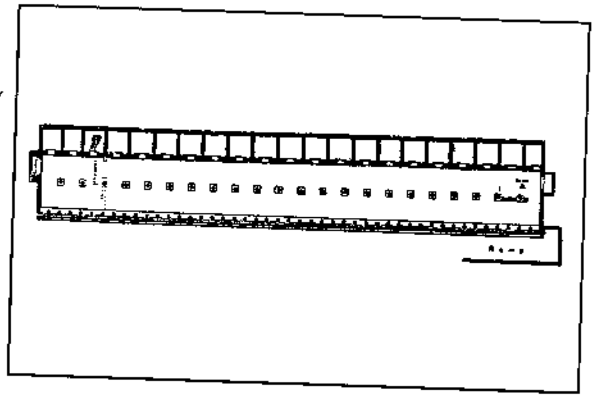
1.7
The Pantheon, Rome, Italy, 1st century A.D.



1.8
The Pantheon, elevation and plan.

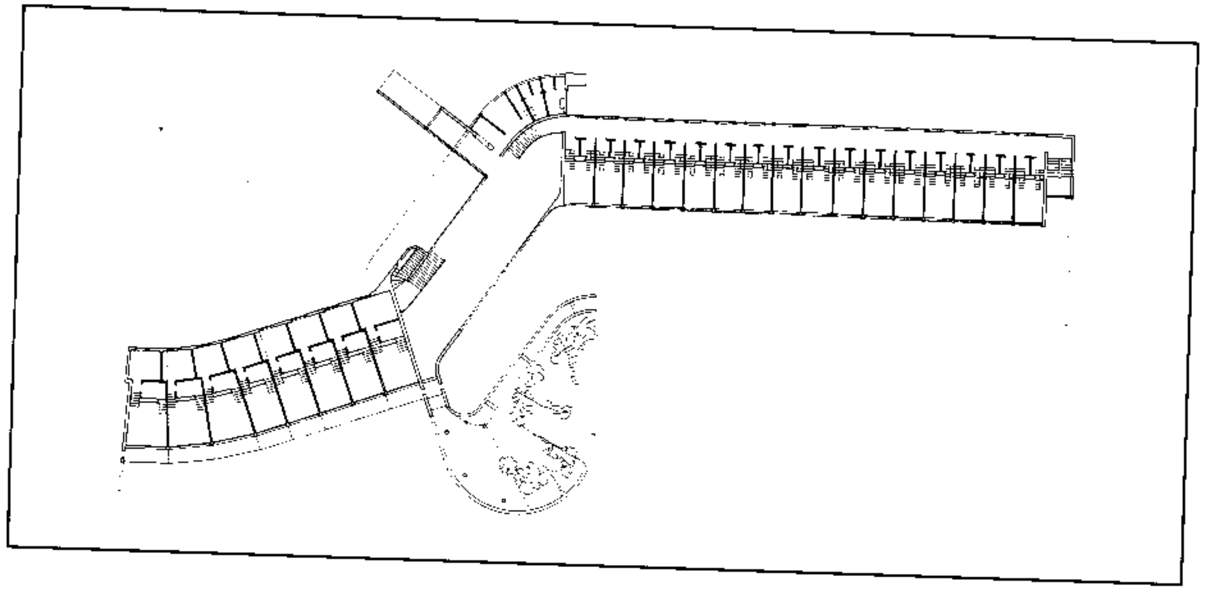


1.9
Stoa of Attalos II, c. 150 B.C. the Agora, Athens, Greece.



1.10
Stoa of Attalos II, plan.

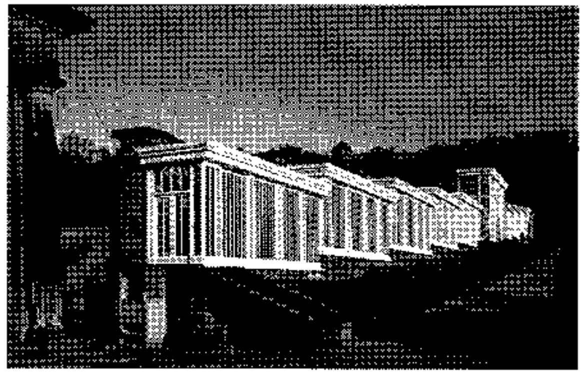
Centralized buildings may grow complex but still make reference to the central point around which they are organized. Linear buildings may also have more elaborate internal configurations. The simplest form of a linear structure is a wall; a bar of matter which is consistent throughout its length, like a bar of chocolate. However, the width of the bar itself may yield a more intricate organization. The space between the two exterior surfaces of a wall, called **poché,** can be carved to shape spatial figures, like the holes formed in a thick mass of Swiss cheese. A linear or 'bar' building can be permeable, like a comb, in which the surface is not continuous but marked by a regular rhythm of vertical members. A bar-building can be cellular, like a Tootsie Roll, in which the line is comprised of like units which are serially repeated. A bar-building can be layered, like a Snickers bar, with one surface treatment on the outside, then another, and another to comprise the whole. A Greek **stoa** is an example such a



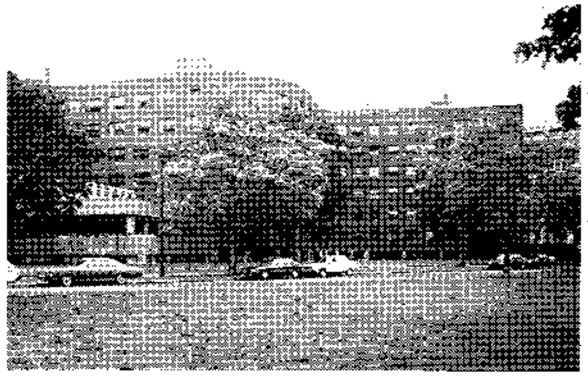
1.11
Hans Scharoun, Hostel, Wroclaw, Poland, 1927.

structure. The side facing the market square or **agora** is columnar in structure. The outer wall, on the contrary, is solid and frequently houses a cellular band of shops. The **Greenhouse at Chatsworth** by **Joseph Paxton** (1801-65), is another example. Attached to a heavy stone wall which girdles the estate is a delicate glass and metal structure. The transparency of the greenhouse enclosure allows both surfaces to be read simultaneously and strongly differentiates the inside from outside.

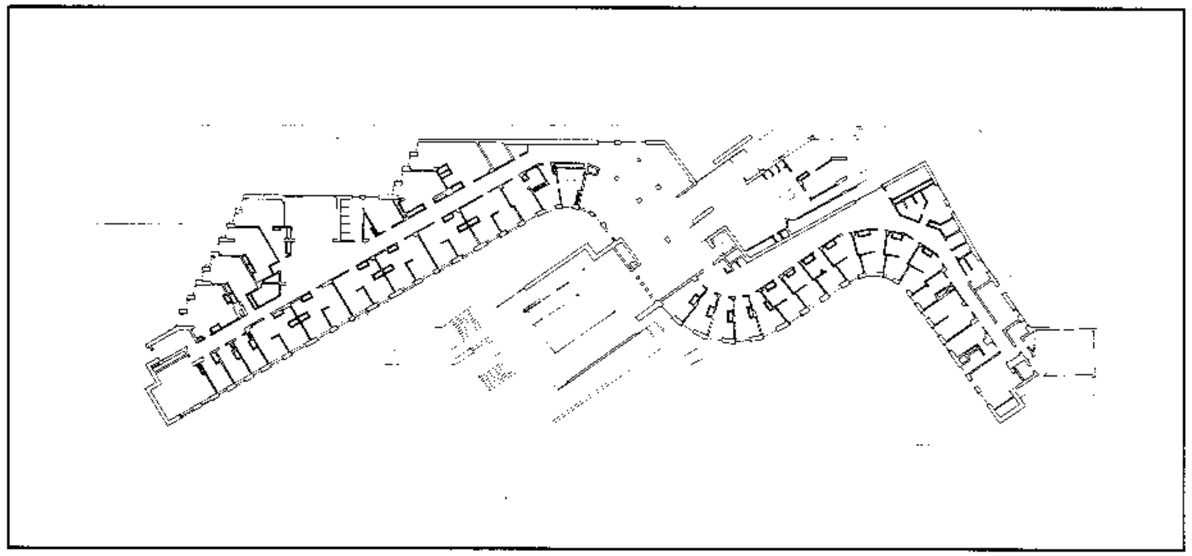
While centralized buildings insist on the equality of all space surrounding them, linear buildings clarify difference from one side of the line to the other. When a line bends or deflects it no longer simply describes a boundary but it begins to define a space. **Alvar Aalto's** (1898-1976) **Baker House Dormitory** in Cambridge, Massachusetts, gently curves in response to the natural form of the Charles River, forming a garden on the river side of the complex. Differences between inside and outside are further



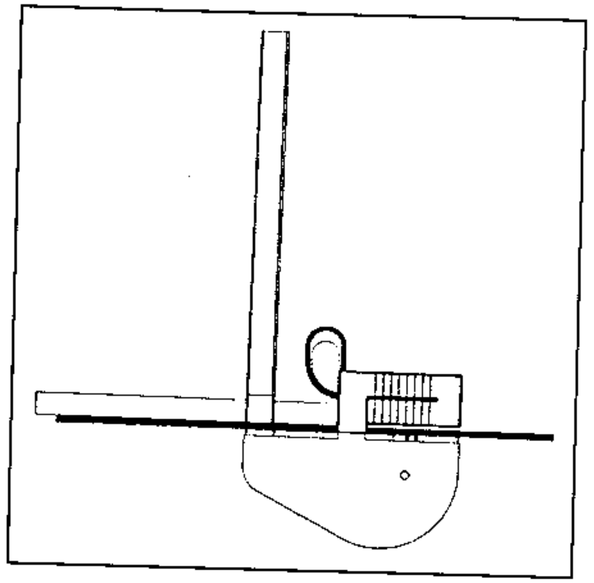
1.12
Joseph Paxton, Chatsworth, England, the Greenhouse, 1836-40.



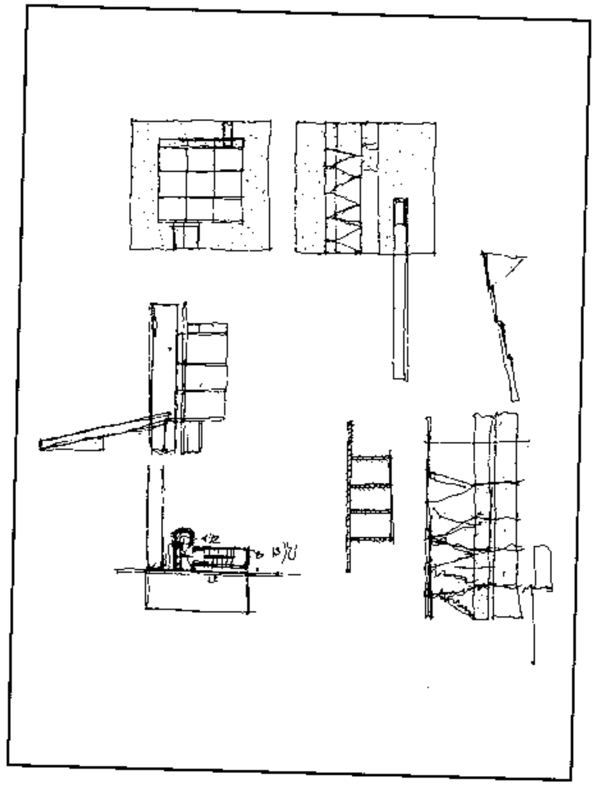
1.13
Alvar Aalto, Baker House Dormitory, Cambridge, Massachusetts, 1947-48.



1.14
Baker House, plan.



1.15
John Hejduk, Wall House I, plan, 1968-74.



1.16
John Hejduk, Wall House I, preliminary sketches.

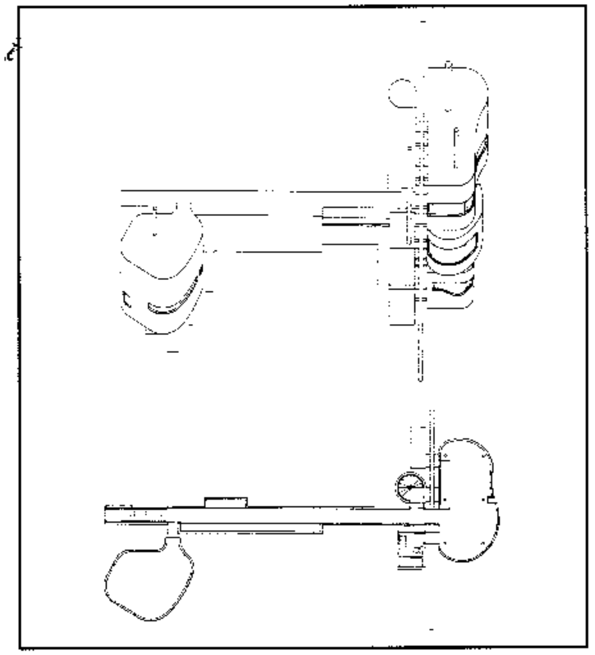
refined by the use of hard, angular forms on the street side in contrast to the gentle undulations of the river side. Moreover, the river side collects private dormitory rooms while the city side is mostly comprised of **circulation**, (corridors, stairs, and elevators,) and collective spaces such as living rooms and bathrooms. At entry, a shaft of space cuts a void through the building, pushing matter out from the bar to form a dining pavilion. Hence, all components of the dormitory derive from transformations of a simple bar.

In **Hans Scharoun's** (1893-1972) **Hostel** in Wroclaw, Poland, the bar of dormitory rooms frames exterior space not so much by bending as by 'twisting.' The open, balconied edge of the long wing threads through the irregularly shaped central hall and reemerges on the opposite side of the short wing, almost as if the particularity of the central space arose through the pressure of the deformation. As in the Baker House, not only form but also **program,** or the functional requirements of a building, are satisfied through this move. Rooms for couples are arranged in the wider bays of the short wing and rooms for bachelors are arranged in the narrower bays of the long wing. The specially shaped central piece houses common living and dining activities.

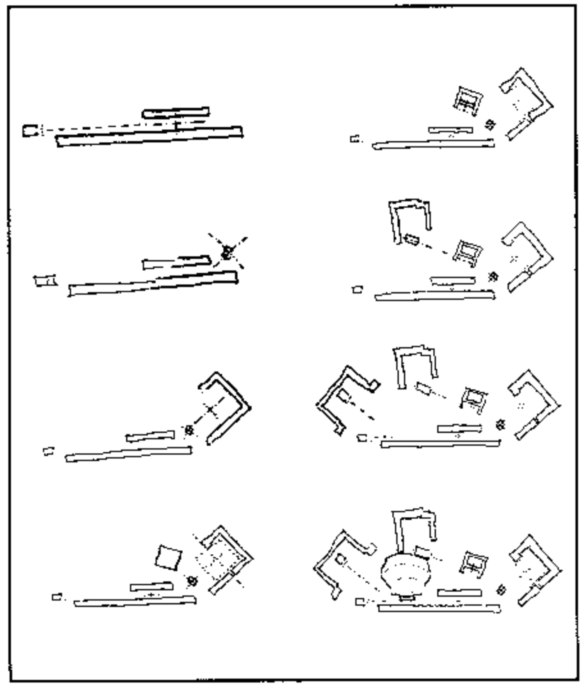
In the previous two examples we observed how a relationship between linear elements and centric objects can arise from a transformative process. In the Baker House, the trajectory of the entry path pushed the rooms at the lower floor outside the bar to form a pavilion piece in the garden. In the Hostel, a twisting of the bar yielded a special condition, much as in the twisting of a Mobius strip. In **Wall House I,**

John Hejduk (b. 1929) reduces the wall to the simple condition of datum. Against the neutral surface of the wall a more particular collection of objects is arranged, not through transformation but through **collage.** The wall emphasizes the difference from one side to the other. Public activities are arranged on one side, private spaces on the other; one side confronts an urban condition, the other side a landscape; one side makes use of a **tectonic,** or constructional system, of heavy, compressive materials, while the other side uses a tectonic system of light, tensile materials, suspended from the wall; one side is comprised of opaque, neutrally colored materials, while the other side is comprised of reflective, transparent surfaces. Hejduk describes his project in the following way: *"on one side of the wall (the past), the circulatory elements— ramp, stair, elevator— were placed. They were volumetric, opaque, monochromatic, in perspective with the structure grounded. The color was white, grey, black; the materials reinforced concrete, steel and cement. Once the single inhabitant passed through the wall he was in a space overlooking a landscape (trees? water? earth? sky?) which was basically private, contemplative and reflective. There were three suspended floors cantilevered from the collective elements. The materials on this side of the wall were glass and reflective metal; a fluidity was sought after. Whereas the collective side was hard, tough, concrete, the private side was inwardly reflective, a light shattering into fragments, mirror images moving along polished surfaces of metal."* (*Mask of Medusa*, by John Hejduk.)

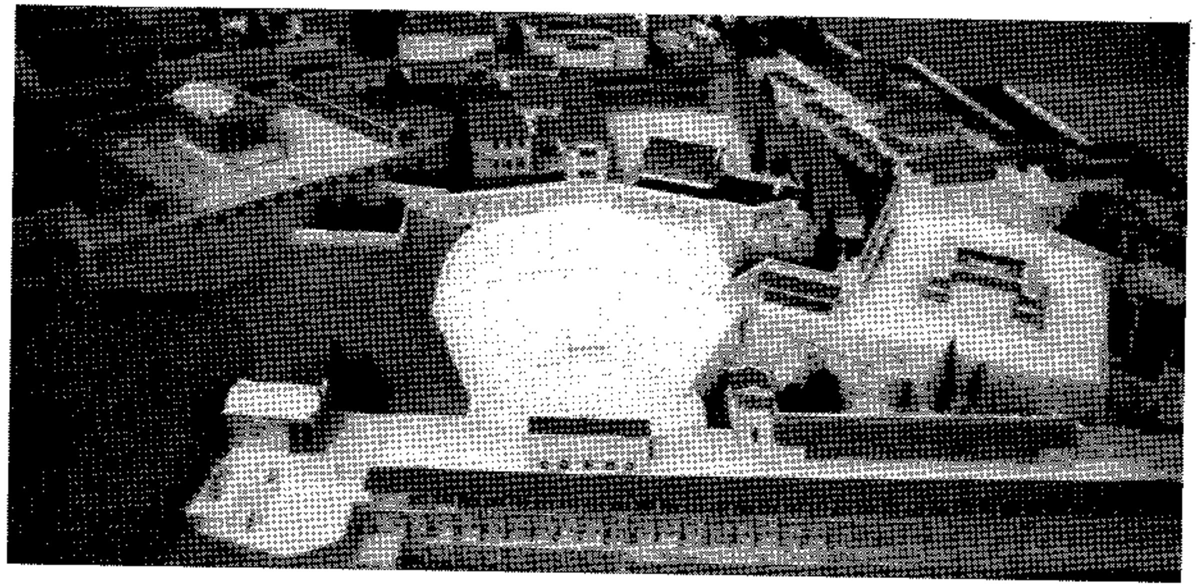
The plan of the Hellenistic town **Pergamon** illustrates an entire context that can be deciphered as a transformation of very simple objects and linear forms to yield a much more



1.17
John Hejduk, Wall House II, axonometric and plan, 1968–74.



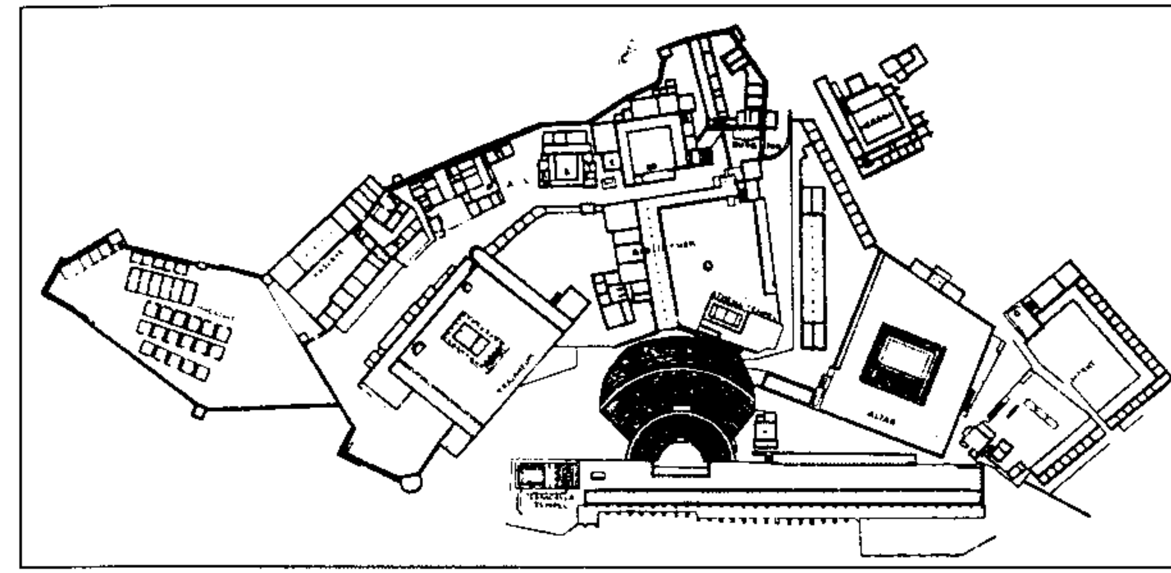
1.18
Douglas Graf, Diagrams of the Upper City, Pergamon.



1.19 Pergamon, Hellenistic.

complex order. In the broadest terms, the urban form of Upper Pergamon can be described as radial, taking advantage of the slope of the hill to carve out the fan-shaped space of the theater, around which secondary spaces splay out like plumes in a fan. A more precise description of interconnections among various spaces in the plan can be construed by imagining acts of formal transformation, just as we earlier read relationships between points, lines, planes and volumes by positing spatial translation from one dimension to the next.

Our reading of the plan begins with the Caracalla Temple at the lower left. The Temple, much like a sugar cube, is a discrete, bounded object, although its permeability along the axis of entry implies extension and organizes a shaft of space in front of it. Just beneath the Temple, a bar-like building, or stoa, extends rightward, as if the temple had been dragged to yield a new linear form. The next element, moving counter-clockwise, is the Market. Here the stoa bends, enveloping in its perimeter the plane of the market area. A few isolated buildings flank the lower edge of the Market, as if the bar had been fragmented to yield discrete objects once again. The plane defined by the Market stoa slips to the left to act as a platform for the Pergamon Altar. Meanwhile, the stoa, which defined perimeter in the Market Complex, shrinks inwards to define the Altar itself. The voided center of the Altar is as bounded and figural as the first term in our progression, the Temple. However, now the figure is defined by an enclosed void rather than a space-displacing solid. While the Temple had a voided perimeter defined by the space of the colonnade and a solid center, the cella, the Pergamon Altar has a solid perimeter and voided center. Hence, form presents itself in its two purest manifestations, as pure solid and pure void. In the Library Complex the bent bar of the Altar splits apart,



1.20 Pergamon, plan.

shattered by the path from the Citadel Gate. In the process, the center is freed from the engirdling wrapper. A monument marks the original location of the center while the true center, the Athena Temple, shifts downwards to align with the central void of the Pergamon Altar to which it formally relates as a figure/ground reversal. The last major space, the Trajaneum, reconstitutes the wrapper and ideal centering of the Pergamon Altar at an expanded scale. However, here a solid object, the temple, marks center. To complete the circuit, one merely needs to slip the temple of the Trajaneum downwards to yield the Caracalla Temple, our starting point.

Reading the plan of Pergamon as a series of transformations performed on fairly straightforward elements makes it possible to decipher order in a plan that at first glance might seem chaotic and without structure. Understanding the various spaces as the product of translations of objects in space, the foldings of edge-defining bars, scale changes, and reversals of figure and ground makes it possible to see how affinities of form and meaning might be constructed. It is worth mentioning that Pergamon did not historically emerge according to an overarching urban strategy of constructing variations on center/perimeter relationships. Pergamon grew up over time, responding not only to the inherent pressures that buildings have upon one another; but also to complicated landform, or **topography**; to precise religious, political and social orderings or **hierarchy;** and to the necessity to make connections to other parts of the town. Still the formal structure yields a 'reading' of the relationship among elements on the site.